# OPTIMIZATION THEORY APPLIED TO THE MODELING OF SANDY BEACH DYNAMICS

## Application to linear seabed

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## INTRODUCTION

Optimization theory is often used to describe physical phenomena in a vast array of domains. Examples include aeronautics, medicine and biology. We wish to use optimization theory to describe the physical phenomena behind coastal morphodynamics.
Unlike other morphodynamic models that depend on in situ and experimental observations, we have a more theoretical approach, only using in situ data for validation purposes.
Continuing the work by (Bouharguane et al., 2010) and (Mohammadi & Bouharguane, 2011), we wish to introduce a new innovative approach to model coastal dynamics, one based on wave-energy minimization.

## OBJECTIVES

- Develop a theory based on optimal control and designed to model the coastal morphodynamics of sandy beaches.
- Implement this new approach to coastal morphodynamics in order to test this theory with numerical simulations.
- Validate the model on a simple case study and show that this new approach to modeling morphodynamics show potential.

## OPTIMIZATION

*“Optimization refers to the minimization or maximization of a certain quantity relative to a given configuration and possibly subjected to constraints.”*

Three elements define an optimization problem:

- A cost function: the quantity to be optimized.
- The input parameters: a set of parameters that can be modified to determine the optimum of the cost function.
- Constraints (optional): restrictions applied to the cost function and input parameters.

## ASSUMPTION

The morphodynamic model is based on the following assumption:

*“The seabed reacts to the state of the waves by minimizing a certain wave-based quantity.”*

We recognize an optimization problem, where :

- the cost function is the wave-based function,
- the input parameter is the shape of the seabed: at each time, the seabed takes the optimal shape, i.e. the one that minimizes the cost function.

## MORPHODYNAMICS BASED ON WAVE ENERGY MINIMIZATION

*What cost function to choose?*

- The minimization of the cost function is considered the driving force behind coastal morphodynamics.
- The choice is important but debatable: it depends on what we consider to be a driving force.
- The cost function should depend on the waves so as to incorporate the undeniable coupling between hydrodynamics and morphodynamics.

➔ We have chosen the energy of shoaling waves:

$$J(\psi,t)=\frac{1}{16}\int_{\Omega_S}\rho_w g H^2(\psi,x,t)\,dx \qquad [J.m^{-1}]$$

water density $[kg.m^{-3}]$; gravitational acceleration $[m.s^{-2}]$; wave height $[m]$; domain over which the waves shoal

*How to find the optimal seabed shape?*

The seabed, in an attempt to minimize the cost function, verifies the following dynamics:

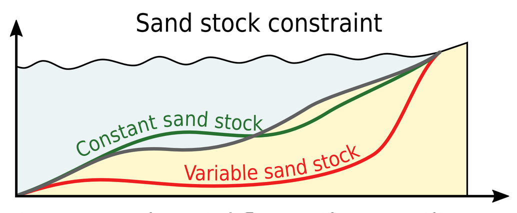
$$\begin{cases}\psi_t = -\rho\,\Lambda\,\vec{d}\\ \psi(t=0)=\psi_0\end{cases}$$

evolution of the seabed; intrinsic sand mobility; influence of water depth; direction of descent: the direction the seabed takes so as to minimize the cost function; initial seabed

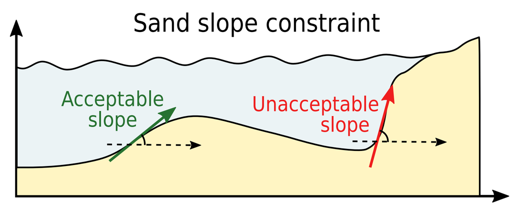
- A natural choice of $\vec{d}$ would be $\vec{d}=\nabla_\psi J$, because $\nabla_\psi J$ indicates a minimum of $J$ with regards to $\psi$.
- Constraints modify the direction of descent $\vec{d}$.

*Do we need constraints?*

- Constraints add more physics to the model.
- They allow us to include processes that aren't driving forces behind coastal morphodynamics but play an important role.
- Two constraints have been included for more realistic results:



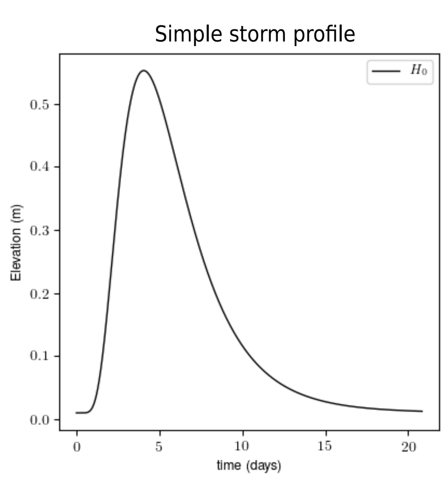
In an experimental flume, the quantity of sand must remain constant over time.

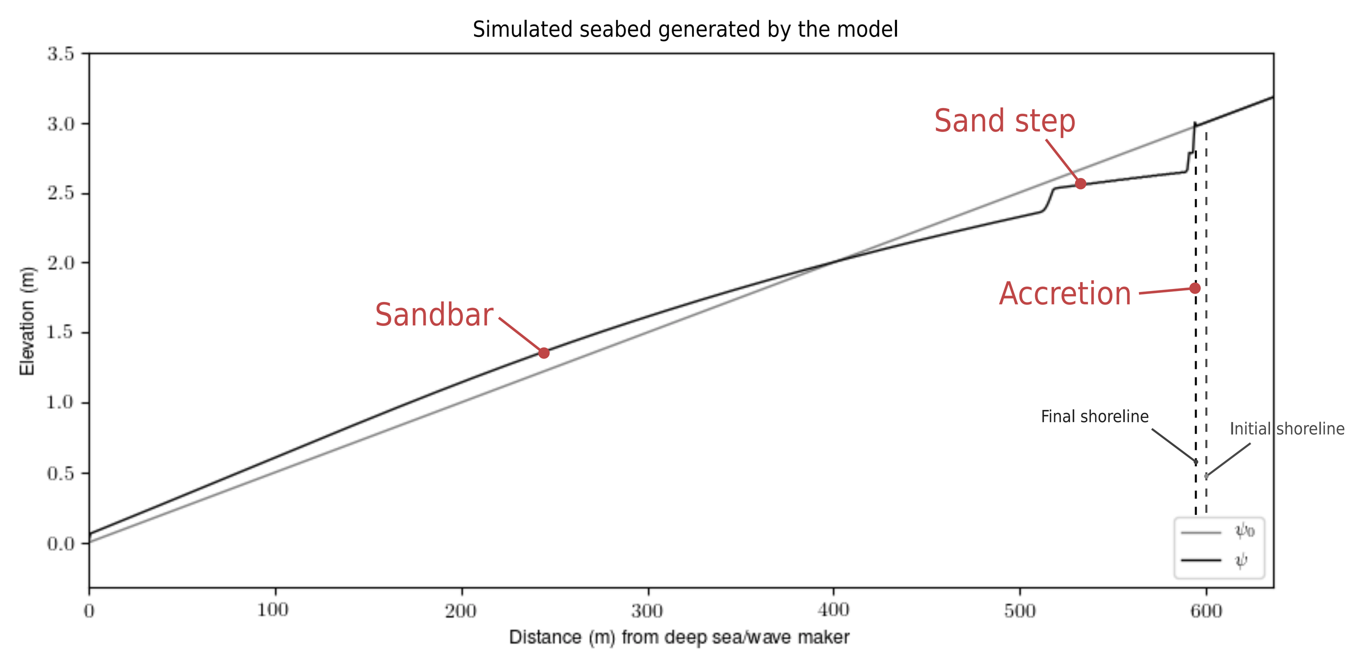


The slope of the seabed cannot be overly steep; the sand would avalanche.

## RESULTS

- We apply the model to a linear seabed.
- We force the system using a simple storm profile.
- We use a basic hydrodynamic model, based on linear wave theory to calculate the height of the waves H mentioned in the definition of $J$.




Three natural phenomena are observed.

- The formation of a sandbar.
- The formation of a sand step.
- Accretion at the shoreline with a small berm.

➔ This morphodynamic model based on wave energy minimization shows potential.

## CONCLUSION

Optimal control is already a valuable tool in coastal engineering, with the design of ports (Isebe, Azerad, Mohammadi & Bouchette, 2008) and defense structures (Isebe, Azerad, Mohammadi & Bouchette, 2008), (Bouharguane, Azerad, Bouchette, Marche & Mohammadi, 2010).
**Can it also be used to model sandy beach dynamics?**
Applying this theory to a simple case study shows that the model generates coastal phenomena observed in situ. We can conclude that this new morphodynamic model based on wave energy minimization shows potential.
Future improvements include the choice of the cost function, the incorporation of additional constraints and the adoption of a more complex hydrodynamic model.

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## REFERENCES

Bouharguane, A., Azerad, P., Bouchette, F., Marche, F., & Mohammadi, B.(2010,06). Low complexity shape optimization and a posteriori high fidelity validation. Discrete and Continuous Dynamical Systems-series B - DISCRETE CONTIN DYN SYS-SER B,13. doi: 10.3934/dcdsb.2010.13.759

Isèbe, D., Azerad, P., Bouchette, F., Ivorra, B., & Mohammadi, B.(2008). Shape optimization of geotextile tubes for sandy beach protection. International Journal for Numerical Methods in Engineering,74(8), 1262-1277. doi:10.1002/nme.2209

Isebe, D., Azerad, P., Mohammadi, B., & Bouchette, F. (2008). Optimal shape design of defense structures for minimizing short wave impact. Coastal Engineering,55(1), 35-46. doi: 10.1016/j.coastaleng.2007.06.006

Mohammadi, B., & Bouchette, F. (2014, 01). Extreme scenarios for the evolution of a soft bed interacting with a fluid using the value at risk of the bed characteristics. Computers and Fluids,89, 7887. doi: 10.1016/j.compfluid.2013.10.021

Mohammadi, B., & Bouharguane, A. (2011, 01). Optimal dynamics of soft shapes in shallow waters. Computers and Fluids, 40, 291-298. doi: 10.1016/j.compfluid.2010.09.031

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