Optimization theory applied to coastal dynamics

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1. Objectives

- Use optimization theory to describe the evolution of the seabed.
- Develop a low-complexity program simulating coastal dynamics by optimization.
- Introduce solid structures (geotubes) used to protect the coast.
- Use optimization theory (again) to determine the optimal position/shape of the structures.

2. Simple hydrodynamic model

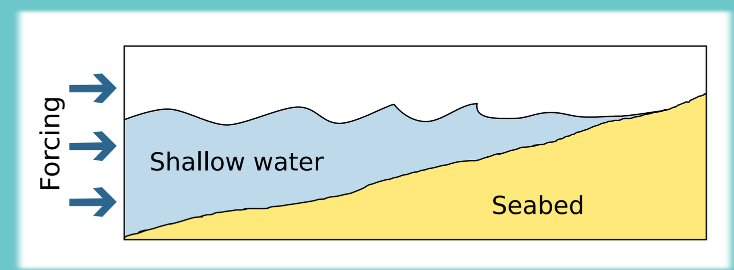
Wave height is defined by:

$$H(x) = \begin{cases} H_0 \left(\frac{1}{2n} \frac{C_0}{C} \right)^{1/2} & \text{if } x \text{ is in the shoaling zone} \\ \gamma \, h(x) & \text{if } x \text{ is in the breaking zone} \end{cases}$$

where:
- H_0 and C_0 are respectively the incoming wave height and velocity
- $n = \frac{C_g}{C}$
- C and C_g are respectively the wave velocity and group velocity
- h is the water depth and γ a wave breaking index

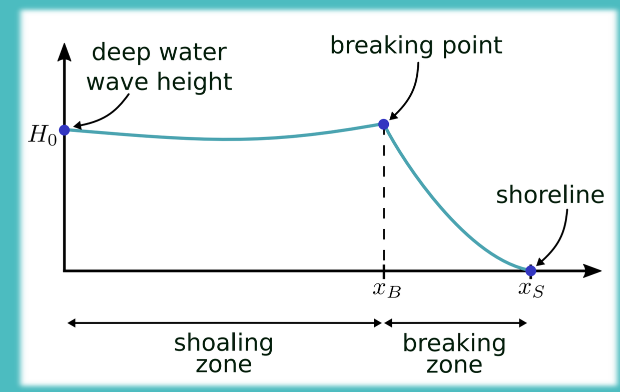
I. Forcing

Provides time-dependent data of the incoming wave.



II. Hydrodynamic model

Provides the morphodynamic model with the hydrodynamic data it requires. Here: the height of the waves.



Assumption:

In the coastal zone, the seabed evolves in order to minimize a wave-related function.

III. Morphodynamic model

Determines the evolution of the seabed based on wave energy minimisation.

The evolution of the seabed is based on the gradient descent method:

$$\psi_t = -\rho \, \Lambda \, \vec{d}$$

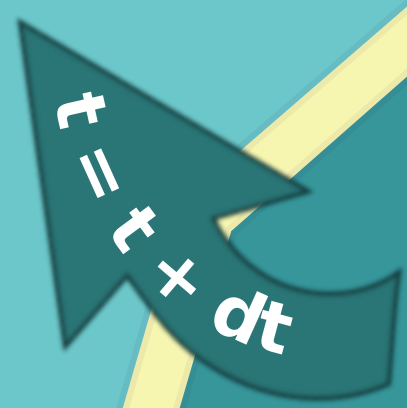
evolution of the seabed (ψ_t); influence of water depth (Λ); intrinsic sand mobility (ρ); direction of descent (driving force) ($\vec{d}$)

A natural choice for $\vec{d}$:

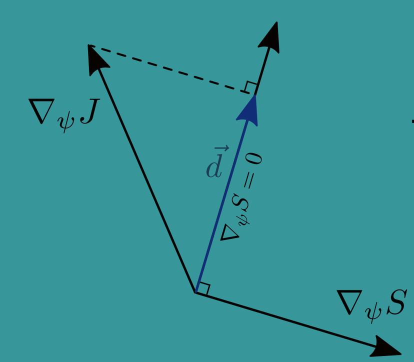
$$\vec{d} = \nabla_\psi J$$

IV. Additional constraints

Adds more physics to the morphodynamic model.



Constraints change the direction of descent $\vec{d}$.

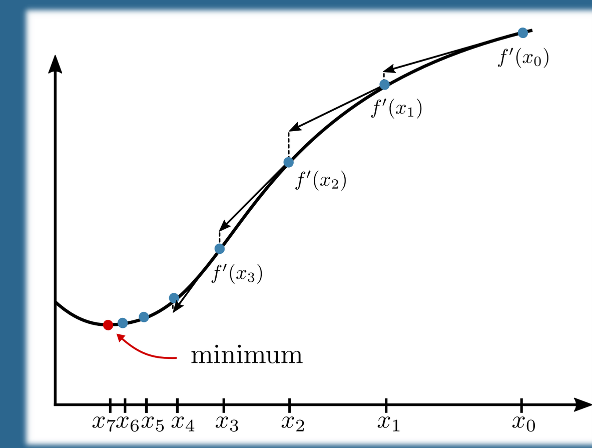


The direction is less optimal but incorporates additional physics to the model.

3. Gradient descent method (1D)

Our direction of descent is based on the gradient descent method.

Let f be a real valued differentiable function of x.



A minimum of f with regards to x is given by the limit of:

$$x_{k+1} = x_k - \alpha f'(x_k)$$

where the step is given by α.

NB: $f'(x)$ indicates a minimum of f.

4. Morphodynamic model

We assume the seabed adapts to minimize the wave-energy function:

$$J = \frac{1}{8} \int_0^{x_B} \rho_w g H(x)^2 dx \quad [J.m^{-1}]$$

where :
- $\rho_w \approx 1000 \ kg.m^{-3}$ is water density
- $g \approx 9.81 \ m.s^{-2}$ is gravitational acceleration
- $H[m]$ is the wave height (provided by the hydrodynamic model)

NB : The choice of J is debatable. It depends on what we consider to be the driving force behind coastal morphodynamics.

A minimum of J with regards to ψ is given by the limit of:

$$\psi_{k+1} = \psi_k - \rho \Lambda \nabla_\psi J$$

where the step is given by $\rho\Lambda$.

NB: $\nabla_\psi J$ indicates a minimum of J with regards to ψ.

The direction on descent $\vec{d}$ is driven by the vector $\nabla_\psi J$.

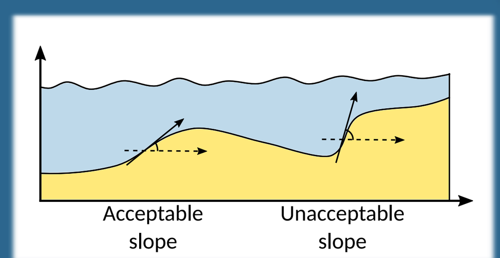
5. Constraints

- In the case of an experimental flume, the quantity of sand remains constant over time:

$$\nabla_\psi S = \nabla_\psi \left(\int_\Omega \psi(x,t) - \psi(x, t=0) dx \right) = 0$$

- The slope of the seabed cannot be too steep:

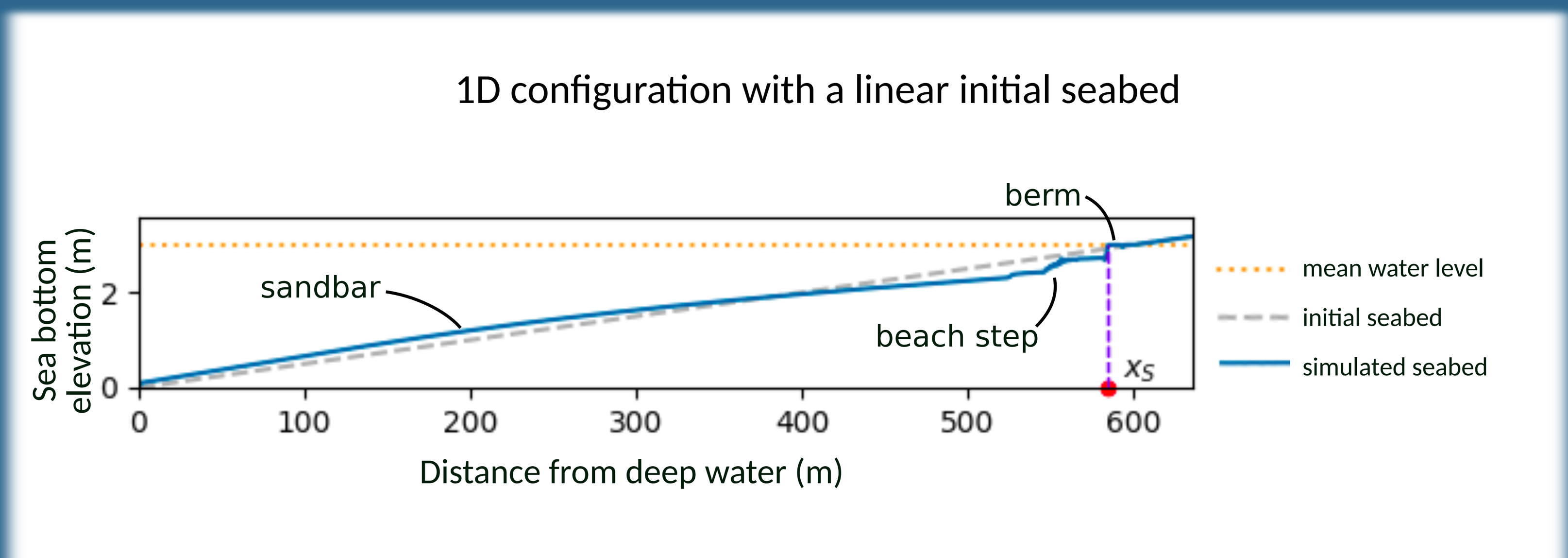
$$\frac{\partial \psi}{\partial x} \leq \beta$$



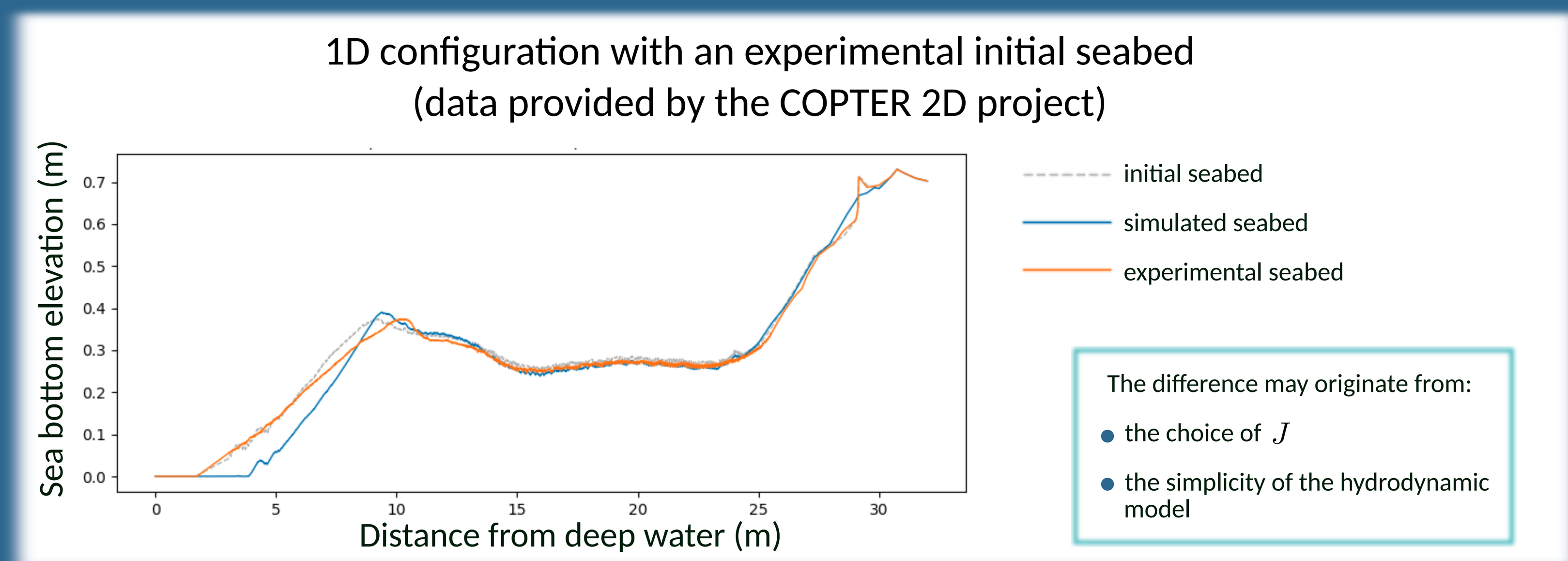
- The movement of the seabed doesn't change the total quantity of energy present in the system:

$$\nabla_\psi \left(\frac{1}{8} \int_\Omega \rho_w g H^2(x) dx \right) = 0$$

6. Numerical results



1D simulations are good representations of experimental flumes.



JDD GM 2019